A Theory of Cost-Sharing Negotiations of Alliances

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The literature on military alliances suggests that alliances can deter aggression through costly signaling. In reality, however, protégés often share substantive alliance costs, which makes alliances cheaper for a patron and should derail the signaling. Why do protégés do that? To explain this gap, I develop a formal model in which allied countries negotiate their cost-sharing under the shadow of international crisis and domestic politics. The model identifies two means by which cost-sharing negotiations sustain peace. First, successful negotiations keep a patron's involvement by reducing the alliance costs when the patron is *not* committed. Second, a large cost-sharing demand makes the negotiations fail, but it signals the patron's commitment and panders to domestic isolationism at the same time. Empirical records of the US-Japan alliance in 1978 and 2019 are explained through these mechanisms.

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1 Introduction

In November 2019, the Trump administration was reported to have asked Tokyo to pay four times as much to offset the costs of US forces stationed in Japan even though it already paid "$2 billion to offset the cost of 54,000 U.S. troops" (Seligman and Gramer, 2019). Along with Trump's similar complaints about the "delinquent" contribution of South Korea and NATO, the administration evoked questions about the United States' commitment to its alliances all over the world (Hille et al., 2020). Trump would be unique in his way of delivering the request, but the disagreement over the cost-sharing between the US and its allies is not new at all. In fact, the past administrations of the US have requested US allies to shoulder more of the costs of their alliances and some of its allies have paid a substantial amount of the costs.

This phenomenon is interesting given that the current scholarly understanding regards alliances as a costly signaling device (Morrow, 1994, 2000; Smith, 1995). The idea is that, if alliances are costly enough for a patron, the patron can signal its intention to help its protégé' if the protégé is attacked. Simply put, a patron who does not invest in its alliance appears to be less committed and fails to deter aggression. If this understanding is true, cost-sharing between allies is puzzling. Sharing the costs of alliances would make them cheaper for a patron, which makes it closer to cheap talk. This should weaken the signal of intervention and increase the uncertainty held by a potential challenger[1]. If the negotiations are about a protégé's arming level and the costs are used for improving the capabilities of a protégé itself, often discussed as burden-sharing in the literature (Iwanami,

[1]Taking reputational costs of abandoning an ally into account would make this question more puzzling. Such costs would increase the threshold of asking cost-sharing to an ally, and thus, it renders a signal of being uncommitted.

2023; Blankenship, 2021, 2023, 2020), the phenomenon that a protégé arms in response to a patron's request is less counter-intuitive since these costs directly improve deterrence, though may not be good for signaling. The cost-sharing I focus on in this paper, however, is different. Japan and South Korea pay for salaries of workers of US bases, utilities bills, and construction fees for US facilities such as houses, golf courses, and movie theaters that are exclusively for US soldiers[2]. It is hard to imagine that these costs directly contribute to enhanced deterrence. Why do some US allies agree to pay the costs of their alliance even though it does not directly improve their power and should weaken the signaling?

Alliance cost-sharing by protégés is even more puzzling given that politicians often claim the positive effect of alliance cost-sharing on their deterrence. For example, right after reaching an agreement about cost-sharing of the US Forces Japan with the Biden administration, a Japanese Foreign Minister expressed his view, " reaching to a necessary agreement at the early stage of the Biden administration shows both countries' strong commitment in the US-Japan alliances, increases the credibility of the alliance, and dispatches it to the international society[3]." A similar view is shared by the United States in its 2017 joint statement, saying the cost-sharing by the Japanese government " serves as a pillar of

[2]For South Korea, see pp. 154-156 of 2022 White Paper of the Ministry of National Defense of Republic of Korea (URL: https://www.mnd.go.kr/cop/pblictn/selectPublicationUser.do?siteId=mndEN&componentId=51&categoryId=0&publicationSeq=1057&pageIndex=1&id=mndEN_0313000000000 Last Access: November 25, 2023). For Japan, see pp. 379 -281 of 2023 White Paper of the Ministry of Defense of Japan (URL: https://www.mod.go.jp/en/publ/w_paper/index.html Last Access: November 25, 2023) and the website of the Ministry of Defense of Japan (URL: https://www.mod.go.jp/j/approach/zaibeigun/us_keihi/ Last Access: November 25, 2023). These costs are called in many ways. They are often called Host Nation Support in US government documents, but it is also called cost-sharing in Japanese or South Korean documents, as well as Omoiyari Yosan (sympathy budget) in Japanese.

[3]Extraordinary Press Conference by Minister Toshimitsu Motegi on February 17th, 2021. Website of Ministry of Foreign Affairs (URL: https://www.mofa.go.jp/mofaj/press/kaiken/kaiken4_001029.html Last Access: November 25, 2023). translated by the author

the Alliance and a symbol of Japan's enduring support for the U.S. military presence in Japan" [4]. If their claims are true, by what mechanism does alliance cost-sharing improve deterrence? The literature does not have an answer to this question.

To account for the questions above, I develop a three-actor formal model in which allies have a cost-sharing negotiation under the shadow of international crisis and domestic politics. Unlike most existing models, I allow endogenous cost-sharing by allies facing a threat of aggression. Specifically, in the model, the alliance cost-sharing affects (a) the costs and survival of the alliance, (b) the reward from domestic constituencies for a patron, and crucially (c) the success of signaling. The patron starts a new cost-sharing negotiation of the alliance with its protégé. This negotiation decides who pays how much, but also it entails the danger of the patron's withdrawal from the alliance. After the negotiation, they enter crisis bargaining with a potential aggressor who is unsure about how much the patron's preference has in common with its protégé's. The potential aggressor observes the result of the negotiation and can change its strategy based on it, so, the allies have to make decisions at the cost-sharing negotiation stage in consideration of the later crisis bargaining.

The model reveals a credible threat of abandonment as key to successful cost-sharing negotiations. A protégé does not agree to share additional costs of the alliance unless a patron's threat of withdrawing from the alliance is credible. Put differently, successful cost-sharing negotiations happen only when the patron would withdraw from the alliance otherwise.

[4] Joint Statement of the Security Consultative Committee, The website of US Department of State (URL: https://2017-2021.state.gov/joint-statement-of-the-security-consultative-committee/index.html Last Access: November 25, 2023)

Based on this insight, the model identifies two means by which cost-sharing negotiations sustain peace. First, successful negotiations keep a patron's involvement by reducing the alliance costs when the patron is *not* committed. Given that successful cost-sharing negotiations occur in equilibrium only when a threat of withdrawal is credible, such success is a sign that the patron does not highly value the issue at stake, and this information is revealed to a potential aggressor. However, a successful cost-sharing negotiation also makes the alliance cheaper for a patron, and this makes it easier for her to maintain the alliance. Keeping the alliance brings a capability boost for the allies, leading to the avoidance of large concessions. In other words, cost-sharing is successful because the capability boost is prioritized to the signaling aspect of the alliance, which implies a potential tension between signaling and a capability boost in alliances. This equilibrium is also consistent with Morrow (1991), which argues that protégés pay autonomy costs to enjoy security benefits in asymmetric alliances. But, unlike Morrow (1991), this equilibrium specifies what "autonomy costs" are and how the exchange rate is determined.

Second, a large cost-sharing demand makes the negotiations fail, but it signals the patron's strong commitment and panders to domestic isolationism at the same time. When a patron is committed, a threat of withdrawal becomes incredible, eliminating a protégé's incentive to accept a cost-sharing offer. Since cost-sharing does not succeed whatever the patron offers, she maximizes her payoff by pandering to her domestic constituencies, which leads to a high cost-sharing demand when they are isolationists. This large demand is beyond the acceptable range for the protégé, and thus, the cost-sharing negotiation is doomed to fail. However, a potential aggressor rationally conjectures that such failure is a sign of the protégé's confidence in its patron's support. Thus, a failed negotiation,

coupled with a large demand, signals the strong commitment of the patron, leading to peace and successful deterrence. This equilibrium suggests that there emerges a *positive* correlation between a patron's commitment and a cost-sharing demand because of the patron's domestic isolationists. In other words, a cost-sharing demand becomes large on equilibrium because of domestic isolationism *and* a patron's strong commitment.

Based on the model, I account for empirical records of cost-sharing negotiations in the US-Japan alliance in 1978, when Japan started the cost-sharing with the United States outside of an existing framework, and in 2019, when Japan rejected a cost-sharing offer from the Trump administration. This article offers the first rigorous theory of alliance cost-sharing.

From the perspective of alliance management, this paper also sheds light on the question of how allies manage to get through changes in their strategic environment and maintain their deterrence. Studies show that large changes in power are strongly associated with dishonoring their alliance commitments (Leeds, 2003a) and changes in preferences are the main reasons for alliance termination (Leeds and Savun, 2007). Descriptive statistics also show that new military alliances have been rare recently and existing military alliances are getting older (Kenwick and McManus, 2021), which suggests that the existing alliances survived changes in their strategic environment such as the end of the Cold War or the Vietnam War. This article explains why some alliances have survived these environmental chances through the cost-sharing mechanisms of alliances. The model shows that, in equilibria, both successful and failed cost-sharing negotiations can be beneficial for deterrence, but in different ways: successful negotiations sustain an alliance, and thus, allies keep high aggregated capability even when a patron is *not* committed. Failed negotiations, despite a

high cost-sharing demand, reveal a protégé's confidence in a patron's strong commitment, especially when a patron's domestic isolationism is strong. This article provides important implications of cost-sharing negotiations in military alliances. Cost-sharing negotiations work as a regulator valve both when a patron is committed and not committed, and thus, cost-sharing negotiations allow alliances to survive longer by overcoming changes in the strategic environment.

2 Literature

The main contribution of this article to the literature is to cast light on the signaling aspect of cost-sharing negotiations between allies. Morrow (1994, 2000) and Smith (1995) all regard alliances as costly signaling and explain why alliances deter aggression through this mechanism. This view is the most canonical and current understanding of alliances, as shown by the fact that many empirical studies and literature reviews employ this as a source of the deterrence effect of alliances (Leeds, 2003b; Johnson and Leeds, 2011; Leeds and Anac, 2005; Morrow, 2000; Kenwick and McManus, 2021). However, these studies do not pay attention to (asymmetric) cost-sharing of alliances, meaning cost-sharing negotiations, especially protégé states' influence in it, are not recognized in the signaling process. By measuring peacetime costs using treaty designs in the ATOP project (Leeds et al., 2002)[5], Johnson (2022) empirically finds that peacetime costs of alliances are larger when allies face a stronger challenger, suggesting that the amount of peacetime costs of alliances are selected "carefully with their adversaries in mind". This is one of the most important papers

[5]To wit, scholars find it not easy to directly measure the costs of alliances, as seen in the discussion between Alley and Fuhrmann (2021) and Cooley et al. (2022).

studying peacetime costs, but peacetime costs are assumed to be determined as a function of a potential challenger, not intra-alliance politics, meaning cost-sharing negotiations between allies are out of the scope. Similarly, Johnson (2015) studies the interaction of a protégé's concession and alliance formation, but does not focus on how existing alliances adjust their cost-sharing after alliance formation and its effect on crisis bargaining.

Other existing studies investigate alliance burden-sharing, not cost-sharing (Blankenship, 2021, 2023; Becker et al., 2023; Blankenship, 2020; Iwanami, 2023). They regard burden-sharing as coordinated military capability among allies and focus on how much allied countries arm themselves in response to their partner's request, sometimes under the shadow of aggression (Iwanami, 2023). However, these studies do not explicitly focus on how signaling can happen in the context of burden-sharing. Although in a slightly different context, this article focuses on the signaling aspect of cost-sharing negotiation by using a formal model, which I will explain in the next section.

3 Model

The model has three actors, Ally (A), Target (T), and Challenger (C). I call A she, T he, and C it to avoid confusion. A and T have an alliance from the beginning of the game and A is supposed to intervene when T is attacked. T and C have a conflict over an issue space that T holds, such as territory, of which value is 1 for both T and C and is $\beta \in (0,1)$ for A. β can be thought of as A's bias over the issue space, or how much A shares T's preference. I assume C is uncertain about A's value of the issue (β). The game starts with Nature (N) randomly choosing A's type; a committed type ($\overline{\beta}$) with the probability of $\phi \in (0,1)$ and

an uncommitted type ($\underline{\beta}$) with the probability of $1-\phi$. This information is revealed to A and T, but not C[6].

Following Morrow (1994), the alliance has two effects. First, the alliance is costly. In the model, the total costs of the alliance are denoted as $\pi > 0$. These costs are necessary to work as costly signaling and can be understood as peacetime costs in Morrow (1994). The costs of the alliance, π, can entail various things, including the restriction of foreign policy autonomy (Morrow, 1991), having a shared military plan in case of war (Poast, 2019; Johnson, 2022), and the costs of overseas military deployment and joint military exercises. Second, the alliance increases the possibility of winning if allies fight together (Morrow, 1994). By keeping an alliance, states find it easier to have integrated joint military plans (Poast, 2019; Johnson, 2022) and gain public support (Tomz and Weeks, 2021; Tomz et al., 2020, 2023). Thus, I assume that A and T win with a higher probability if they keep their alliance and fight together. Specifically, the probability of winning is denoted as $p \in (0,1)$, and it takes p_l if T fights alone (For example, Poland fought alone against Nazi Germany in 1939, even though it had an alliance with France and the United Kingdom), p_m if T and A fight together without the alliance (The United States and Kuwait fight together against Iraq in 1990 without an alliance), and p_h if they fight together with the alliance (Germany and Austria-Hungary had an alliance and fought together against Russia in 1914). Note that $p_l < p_m < p_h$. The difference between p_m and p_h is called a capability boost of an alliance.

This model has two stages; the cost-sharing negotiation stage and the crisis bargaining stage. Figure1 shows the cost-sharing negotiation stage. In this stage, after A's type is

[6]No uncertainty between allies is a common assumption in the literature such as Fang et al. (2014). See Smith (2021) for a model where coalition partners have uncertainty with each other.

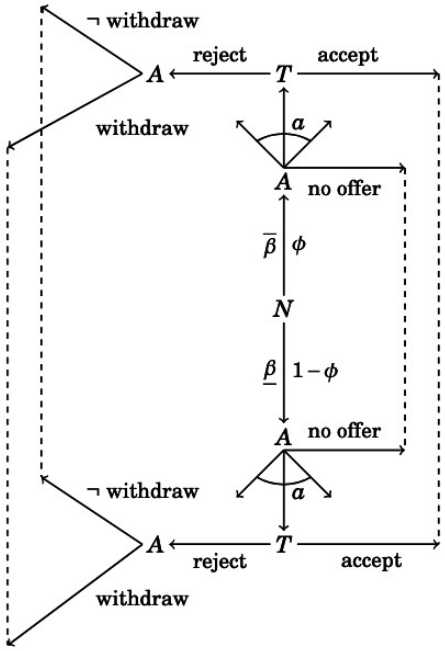

Figure 1: Cost-Sharing Negotiation Stage

realized, she chooses to make either "no offer" or a cost-sharing offer. In the former case, the cost-sharing negotiation does not happen, and A pays the full costs of the alliance (π), meaning no change in the cost-sharing. In the latter case, she demands some cost-sharing by T, and the amount of the offer is denoted as $a \in (0, \overline{a}]$, where $\overline{a} > 0$ is the upper limit of T's ability to pay[7].

I do not presume the way a is paid, but the prime example is Japan and South Korea's monetary contribution to the US forces stationed in each country. Then, T decides whether to accept the offer. Accepting the offer is a success of the cost-sharing negotiation and this imposes a on T and $\pi - a$ on A. Rejecting the offer is a failure of the negotiation. This does not impose any costs on T but gives A an opportunity to withdraw from the alliance.

[7] I tolerate that $\overline{a}$ takes a larger value than π to allow the possibility that T is exploited more than the total costs of the alliance. The equilibria shown in this chapter exist even when I use π as an upper limit.

If A withdraws from the alliance, both A and T do not pay any costs, but they cannot benefit from the alliance capability boost[8]. If A does not withdraw from the alliance, she pays the full cost of the alliance, π. In sum, the alliance cost-sharing negotiation has four outcomes: no cost-sharing negotiation, which I call "no offer", the success of the negotiation, which is "sharing", the failure of the negotiation with A remaining in the alliance, which is "free-riding", and the failure of the negotiation without the alliance, which is "withdrawal".

Finally, I incorporate elements of domestic politics by allowing A to get la when she demands something in the negotiation. The domestic politics term, $l \in (-1, 1)$, represents the level of domestic isolationism/internationalism in A, and it brings punishments/rewards for a harsh attitude towards its ally. When $l > 0$, the domestic constituencies are isolationists on average and reward a leader's harsh attitude towards allies by giving a positive value ($la > 0$) to the leader, for example, by re-electing the leader with a higher probability. When $l < 0$, they are internationalists and punish a leader who demands a lot from their allies by giving a negative value (e.g. not re-electing the leader)[9]. The upper and lower limits of l ease the analysis, but substantively, it assumes that the material benefits directly from the cost-sharing negotiation are always larger than benefits/costs from domestic politics

[8]The interpretation of the withdrawal from the alliance is flexible, including many things from alliance termination to partial withdrawal. This does not change the implication of the model as far as it loses a capability boost of an alliance.

[9]Note that these rewards/punishments are for the *attitude* of the leader, not for the result of the negotiation itself. Harsh attitudes towards the cost-sharing issue, even if they are not successful, correlate with the leader's attitude toward other diplomatic issues such as international organizations or trade barriers, and I assume that the domestic constituencies reward or punish these attitudes to discipline leaders' future behavior. Alternatively, we can interpret l as a leader's personality that gives the leader a positive or negative value from demanding a lot from an ally, and domestic constituencies elect a leader that fits their preference. This does not change the interpretation of the results. In any case, this way of representing domestic politics is more appropriate than hand-tying in the context where strong partisanship immunizes a leader from a negotiation failure or inconsistency (See McDonald et al., 2019).

($a > |la|$ for $l \in (-1, 1)$), which makes the benefits from the cost-sharing negotiation is a priority for a leader. Otherwise, domestic politics would dominate the cost-sharing negotiation and the international negotiations would become meaningless.

I denote the payoffs resulting from the cost-sharing stage as $\lambda_{i,n}$, where i is the player and can take either A or T ($i = \{A, T\}$) and n is the result of the negotiation and can take one of the four outcomes of the negotiation ($n = \{\text{no offer}, \text{sharing}, \text{free-riding}, \text{withdrawal}\}$). The payoffs from the negotiations are formally expressed below.

$$\lambda_{A,n} = \begin{cases} -\pi \quad (\text{if } n = \text{no offer}) \\ -\pi + a + la \quad (\text{if } n = \text{sharing}) \\ -\pi + la \quad (\text{if } n = \text{free-riding}) \\ 0 + la \quad (\text{if } n = \text{withdrawal}) \end{cases} \tag{1}$$

$$\lambda_{T,n} = \begin{cases} -a \quad (\text{if } n = \text{sharing}) \\ 0 \quad (\text{if } n = \text{free-riding, no offer, or withdrawal}) \end{cases} \tag{2}$$

Next, the model moves on to the crisis bargaining stage. In this stage, C first proposes a division of the one-unit policy space to T, which is expressed as $x \in (0, 1)$, where x is what T keeps and $1 - x$ is what C obtains. After receiving the demand offer, T choose to concede or resist. If it concedes, the game ends with settlement and C's offer is realized. If T rejects, a war between T and C breaks out. Then, A chooses whether to help T or not. If A helps T, the war expands to a multinational war. If A does not help, then the war remains bilateral. A war is expressed as a costly lottery, and actors have to pay the war costs (c_i) if they are involved. The final payoffs are shown below.

$$u_i(\text{Settlement}) = \begin{cases} \beta x + \lambda_{A,n} & (\text{if } i = A) \\ x + \lambda_{T,n} & (\text{if } i = T) \\ 1 - x & (\text{if } i = C) \end{cases} \tag{3}$$

$$u_i(\text{Bilateral War}) = \begin{cases} \beta p_l + \lambda_{A,n} & (\text{if } i = A) \\ p_l - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_l - c_C & (\text{if } i = C) \end{cases} \tag{4}$$

$$u_i(\text{Multilateral War w/ alliance}) = \begin{cases} \beta p_h - c_A + \lambda_{A,n} & (\text{if } i = A) \\ p_h - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_h - c_C & (\text{if } i = C) \end{cases} \tag{5}$$

$$u_i(\text{Multilateral War w/o alliance}) = \begin{cases} \beta p_m - c_A + \lambda_{A,n} & (\text{if } i = A) \\ p_m - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_m - c_C & (\text{if } i = C) \end{cases} \tag{6}$$

4 Key Features of the model

The model has some key features. First, building on existing works, the costs of alliances are sunk (Morrow, 1994, 2000). Scholars often regard the formalization as the source of deterrence effects of alliances, and the reason why states "write them down" and make them open to the public is that states can generate some sunk costs (Morrow, 2000), which work as costly signaling (Fearon, 1997). In my model, A and T have to pay the costs of the alliance, if it is not abandoned, regardless of the results of the crisis bargaining.

Second, the depth of cost-sharing (a) does not directly affect military capabilities. This is because it is hard to imagine an automatic relationship between the alliance costs T pays and the improved military capability, where the costs are utility bills, houses, amusement facilities, and so on. If there is such a relationship, there must be some political process over the alliance costs and the military capability between allies, and this is exactly what the model tries to capture.

Third, the total costs of the alliance, π, are given and fixed in the model. Scholars often assume that the alliance costs such as peacetime costs or the level of institutionalization are determined at the alliance formation stage (Leeds and Anac, 2005; Johnson, 2022; Morrow, 1994). As my focus is on how allies manage the costs of their *existing* alliance, I set π as a fixed value. One can interpret this parameter as the optimal level of the total costs of the alliance to deter an aggressor or fixed costs of the investment in international institutional building, but allies have a conflict over who pays how much.

The model is different from existing formal models in the following points. Morrow (1994) investigates how the peacetime costs of alliances, coupled with improved military performance, affect deterrence. But the model assumes that a patron and a protégé pay the same amount of peacetime costs and these costs are not negotiable, meaning the politics of the cost-sharing negotiations, as well as its influence on deterrence, is excluded. Johnson (2015) studies endogenous concessions from a protégé and alliance formation, but the challenger does not have the uncertainty over the patron's intervention, which means that the study is silent on the signaling aspect of the cost-sharing negotiations. Fang et al. (2014)'s model also deals with how alliance affects crisis bargaining, but their focus is on how a partner constrains its protégé and no attention is paid to how the cost-sharing

affects the course of bargaining. Iwanami (2023) frames burden-sharing as "the amount of armed forces each ally deploys" and then researches how much allies individually arm themselves in a complete-information environment, but the cost transfers between allies and their signaling aspects are not discussed. Johnson and Wolford (2023)'s model investigates the effect of alliances on dispute escalations, but a patron and a protégé do not have negotiations, meaning intra-alliance politics is out of their scope. Fang and Ramsay (2010)'s model studies how NATO countries share a burden to contribute to a public good, but their model does not consider a potential opponent's reaction, and thus, does not take into account its effect on crisis bargaining and its feedback effect on the intra-alliance politics.

5 Analysis

My solution concept is Perfect Bayesian Equilibrium (PBE), in which strategies are sequentially rational and consistent with beliefs updated according to Bayes' Rule whenever possible. First, I focus on the following parameters.

Assumption 1.

$$p_l > c_T \tag{7}$$

$$\min\{\overline{\beta}(p_m - p_l), \underline{\beta}(p_h - p_l)\} > c_A > \underline{\beta}(p_m - p_l) \tag{8}$$

Line 7 ensures that there is a certain range of x that T prefers war to settlement even

if he has to fight alone. In other words, T credibly says that he would rather go to war if he has to accept a tiny split of the issue, which makes C always think about what range of x is acceptable for T. Line 8 specifies the range of c_A so that A does not always help or always abandon T. Specifically, under this assumption, the committed A helps T even when they do not have the alliance, but the uncommitted A helps T if and only if she keeps the alliance. This means that A's withdrawal from the alliance is dangerous in two ways. First, the lack of the alliance capability boost may provoke C and push it to demand more in crisis bargaining. Second, the withdrawal induces uncertainty over A's involvement in a war. Since A does not help T if she is uncommitted but it does so if she is committed, C has to guess A's type especially when A withdraws from the alliance.

Proposition 1 (Separating 1)**.** *When assumption 1 is satisfied and*

$$l < 0 \tag{9}$$

$$\overline{\beta} > \frac{\underline{\beta}(p_h - p_l) - c_A}{p_h - p_m - c_T} \tag{10}$$

$$\frac{p_h - p_m - c_T + c_A}{p_h - p_l} > \underline{\beta} \tag{11}$$

$$p_h - p_m > c_T, \tag{12}$$

$$\underline{\beta}(p_h - p_l) < \pi < \overline{\beta}(p_h - p_m - c_T) + c_A, \tag{13}$$

there exists a separating PBE at which the committed type of A does not make any cost-sharing offer, C offers $x = p_h - c_T$, and T accepts it, and the uncommitted A offers $a = \min\{a^* = p_h - p_l, \overline{a}\}$, T accepts the offer, C offers $x = p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 1 is a separating equilibrium in which the committed A does not make any cost-sharing offers, whereas the uncommitted A offers $a = \min\{a^*, \overline{a}\}$ and T accepts the offer. The alliance is sustained and war does not happen in both cases. Since the committed

and uncommitted A behave differently before crisis bargaining, C (correctly) updates its beliefs. This equilibrium exists when the domestic constituencies are internationalists on average (Line 9), the difference between A's types, and thus the uncertainty of A's types, is large (Line 10 and 11), the capability boost of the alliance ($p_l - p_m$) is large enough (Line 12), and the total costs of the alliance is neither too small nor too large (Line 13).

The last condition (Line 13) stipulates important off-equilibrium behaviors of A. Due to the relatively large costs of the alliance (see lower bound of π in Line 13), the uncommitted A would withdraw from the alliance if T rejects a cost-sharing offer. In other words, the threat of abandonment is credible for T when A is not committed. This off-path behavior incentivizes T to accept the cost-sharing offer in order to keep the alliance and enjoy the capability boost (Line 12). However, π is so small (see the upper bound of π in Line 11) that the committed A would sustain the alliance even when T rejects the offer. So, A cannot credibly threaten that she would abandon T if the cost-sharing offer does not succeed. Because of this, T will be better off rejecting the offer since he can enjoy the benefit of the alliance without paying any extra costs. On such an occasion, the committed A cannot get anything from the cost-sharing negotiation, and thus, does not demand anything from the beginning to avoid domestic punishment by internationalists as shown in Line 9.

This equilibrium suggests several significant implications for cost-sharing between allies. First, a credible threat of abandonment is key for successful cost-sharing negotiations. As discussed above, the incentive to accept a cost-sharing offer emerges when T faces a real threat of abandonment and T needs to lower the costs of A's involvement. However, T does not agree to pay if A would not leave the alliance anyway, as in the case of the committed A. A credible threat of abandonment is necessary for success in cost-sharing.

This leads to the next implication.

Second, a successful cost-sharing negotiation is a sign that A is not committed. T agrees on the cost-sharing offer only when A is uncommitted and unwilling to keep the alliance. This, in turn, updates C's beliefs after observing a successful cost-sharing negotiation. C thinks that A that achieves successful cost-sharing is an uncommitted type. In other words, T's acceptance of a cost-sharing offer signals his doubt in A's support. T agrees to pay even though it signals the uncommitted A simply because the advantage of keeping the alliance is large enough thanks to the capability boost of the alliance (Line 12). In more general words, this equilibrium suggests that allies sometimes sustain their alliance even though it ruins the signaling aspect of the alliance. T prolongs the life of his alliance by decreasing the alliance costs for A even though it signals the weak commitment of A. The main reason for this is that A and T can enjoy a good share of the issue and keep high deterrence ($x = p_h - c_T$) as long as they sustain the alliance. The literature often emphasizes the signaling aspect of alliance (Morrow, 1994, 2000; Smith, 1995; Kenwick and McManus, 2021), but this equilibrium implies that alliances can servive even after they lose the signaling feature.

Third, cost-sharing is an exchange between money and power. In the equilibrium, T pays the smaller amount between $\overline{a}$ (the maximum amount of its ability to pay) and $a^* = p_h - p_l$. When T's ability to pay is high enough ($\overline{a} > a^*$), he has to pay a^*, and this is exactly the difference in the value brought by sustaining A's involvement through the alliance. Let's take a look at T's off-equilibrium-path behavior. If T did not accept the cost-sharing offer, A would withdraw from the alliance. Then, the share of the issue for T would be $x = p_l - c_T$ since C demands a lot and T has to accept it due to the lack

of the alliance. The share of the issue on the equilibrium, however, is $x = p_h - c_T$. The improvement of the decision brought by the alliance for T is $(p_h - c_T) - (p_l - c_T) = p_h - p_l$, which is exactly the same as a^*. T buys the enhanced power and enjoys a better share of the issue by paying the same value of money. This is consistent with Morrow (1991), which argues that allies pay autonomy costs to enjoy the benefit of alliance in asymmetric alliances. But, unlike Morrow (1991), this equilibrium specifies what "autonomy costs" are and shows how the exchange rate is determined. In a practical sense, this implies that the larger power an alliance brings, the more money a protégé has to pay. In fact, Japan and South Korea have many US bases and soldiers in their countries and pay a lot to the US, whereas the Philippines does not have such large US bases and has not paid similar costs in history.

The last feature of this equilibrium is a discrepancy between an ostensibly good division of the issue and the actual payoff for T. As discussed in the previous paragraph, the division of the issue on equilibrium when A is uncommitted is $x = p_h - c_T$, which is the same as the division when A is committed. However, since T burdens some of the costs of the alliance, the final payoff for T when its ability to pay is large enough is $p_h - c_T - a^* = p_l - c_T$, which is indifferent from the payoff when T rejects the cost-sharing offer and loses the alliance. Thus, this equilibrium is featured with (a) an ostensibly good division of the issue and (b) lower payoffs for T. To wit, although the model does not talk anything about T's domestic politics, left-wing parties in Japan have strongly criticized the cost-sharing by their government. This may be because of the lack of diplomatic senses or a partisan attitude nested in the Cold War structure, but this surely suggests that elements of the people think that the payment is too expensive and unreasonable. Socialist and communist

parties in Japan even claim the abolishment of the US-Japan alliance, which is consistent with the implication that the utility of the lack of the alliance is almost the same as the utility of the existence of the alliance with cost-sharing.

Proposition 2 (Separating 2)**.** *When assumption 1 and Line 10, 11, 12, and 13 are satisfied and*

$$\overline{a} > a^* \tag{14}$$

$$\min\{1, l^* = \frac{(1+\beta)a^* - \pi}{\overline{a} - a^*}\} > l \geq 0 \tag{15}$$

there exists a separating PBE at which the committed type of A offers $a = \overline{a}$, T rejects it, A does not withdraw from the alliance, C offers $x = p_h - c_T$, and T accepts it, and the uncommitted A offers $a = a^*$, T accepts the offer, C offers $x = p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 2 is a separating equilibrium where the committed A offers $a = \overline{a}$, T rejects it, and A does not withdraw from the alliance. Here, the cost-sharing negotiation fails but the alliance is sustained. The uncommitted A, however, offers $a = a^*$, T accepts it, and thus, the cost-sharing negotiation is successful and they keep the alliance. C correctly updates its belief since the course of the cost-sharing negotiation is different depending on the types of A. Proposition 2 exists under similar parameters of Proposition 1. It exists when the uncertainty is large (Line 10 and 11), the capability boost of the alliance is large enough (Line 12), and π is not too large but not too small (Line 13). The key difference from Proposition 1 is the equilibrium behavior for the committed A, which is driven by the strong but not too strong domestic isolationism in A (See Line 15).

When A is committed, it offers the maximum level of cost-sharing ($a = \overline{a}$). Since the committed A highly evaluates the issue at stake, she cannot credibly threaten that she would

withdraw from the alliance if the negotiation fails. Knowing this, T does not agree to pay even a very small amount of the cost-sharing demand, and the cost-sharing negotiation is doomed to fail. Thus, the committed A cannot get anything directly from the negotiation due to her strong commitment. On that occasion, A maximizes her payoffs by offering the maximum level of cost-sharing demand and pandering to her domestic isolationism (See $l > 0$ in Line 15)[10]. Note that, in Proposition 1, the committed A makes "no offer" because of the internationalism in her country (Line9), but this is not the case in Proposition 2. This is the main difference from the previous equilibrium.

Proposition 2 is interesting because it shows a *positive* correlation between a high resolve of A and a large demand for cost-sharing. When A is committed ($\overline{\beta}$), she offers $a = \overline{a}$. When A is *not* committed ($\underline{\beta}$), she offers $a = a^*$, which is strictly lower than $\overline{a}$ (Line 14). In other words, the more committed A is, the more she demands. This *positive* correlation occurs in equilibrium due to the combination of domestic isolationism and the lack of credible threat of abandonment. As explained above, A's high resolve makes the threat of abandonment not credible for T, which leads to no concession for A in the cost-sharing negotiation. Given this, A demands as much as possible to pander to the domestic isolationists knowing that the negotiation would probably not succeed. The uncommitted A, however, does not demand a lot despite the same level of domestic isolationism because she can get a tangible benefit from the negotiation. Such A offers a lower cost-sharing demand than the committed A and gets the benefits from both the cost-sharing negotiation and partially, if not completely, satisfied domestic constituencies. The strong but not too

[10]We could assume a different (or more general) function that maps a cost-sharing offer to benefit from domestic politics, but this would not change the interpretation of the results: When the domestic constituencies are isolationists, the committed A demands a large and optimal cost-sharing offer and pander to the domestic constituencies knowing that the demand will be rejected by T.

strong domestic isolationism helps A achieve both types of benefit. This is the reason for a high demand for the committed type and a lower demand for the uncommitted type.

An alternative interpretation of this result is that domestic isolationism can appear in the cost-sharing negotiation only when the patron is committed. When the patron is not committed and her domestic constituencies are isolationists, she offers the same cost-sharing demand as a patron that has domestic internationalism ($a = a^*$. See Proposition 1), implying an inability to identify the domestic situations based on international negotiation when a patron is not committed. On the contrary, when a patron is committed, domestic isolationism enlarges the cost-sharing demand ($a = \overline{a}$), and domestic internationalism eliminates such negotiations from the empirical record because a patron does not offer anything in the first place (see, again, Proposition 1). This suggests that domestic politics is reflected in international cost-sharing negotiations when a patron's commitment is strong.

Proposition 2 also shows that the failure of the cost-sharing negotiation occurs when A is committed despite the high cost-sharing demand. When C sees the failure of the negotiation, it infers that this is a sign of T's confidence in A's support. In other words, when C observes that A could not achieve a concession from T, it thinks that T rejected the cost-sharing offer because A would not withdraw from the alliance. And because A would not withdraw from the alliance, C thinks that she must be a committed type. On such an occasion, the large cost-sharing offer makes sense for C given A's commonly-known high domestic isolationism. As such, the failure of cost-sharing negotiation fails due to A's large offer, but this failure signals A's strong commitment and panders the domestic isolationism at the same time. This is another way of achieving peace through cost-sharing negotiations.

This result may sound counterintuitive, but this is still within the conventional costly-signaling mechanism. The failure of cost-sharing negotiations happens when A is fine with *continuing* to pay the full costs of the alliance, whereas the success of the negotiation means that the alliance becomes *less* costly for A. Thus, the failure of the negotiation is more costly for A than the success of the negotiation, and thus, the failure ends up revealing the high resolve of A. This result is consistent with recent research arguing that free-riding by a protégé on a patron's military capability rather signals the patron's high resolve (Smith and Dong, 2024)[11].

In reality, when people see a large cost-sharing demand, they worry that it is a harbinger of fragile alliances and an unstable world. This worry is probably valid in that it is a sign of strong domestic isolationism, which could lead to disagreements between allies and perhaps the termination of an alliance in the long run. But Proposition 2 shows that a large cost-sharing demand panders to domestic isolationism, which may open up a way to prolong the life of the alliance by satisfying the domestic constituency. Also, the model shows that a cost-sharing demand is *amplified due to the strong commitment of the patron and the protégé's confidence in its support*. If a patron is not committed, the cost-sharing negotiation succeeds with a relatively smaller demand, since a patron tries to win both international concession and domestic support and thus has an incentive to lower the demand to make it easier for its protégé to accept the demand. When the patron is committed, this incentive does not work. The protégé loses the incentive to accept even a very small demand, and this rather increases the patron's demand to secure the domestic

[11] Becker et al. (2023) recently reported that US president's negative rhetoric does not increase (rather decreases) NOTO countries' military contribution. Proposition 2 and Smith and Dong (2024) answer why the US allows such free-riding and whether it has a positive or negative influence on deterrence.

benefit.

This viewpoint provides an alternative interpretation of the recent conflicts over the costs of alliances. For example, the US often complains that NATO countries contribute to their alliance at a lower level than their potential capability. The US and its NATO allies seem to have no negotiation range, but they have an incentive to exaggerate their limited room for concession to achieve the other's concession. Moreover, the model implies that such conflicts are not bad in all aspects. Such conflicts can help keep the high level of deterrence of the alliance by revealing NATO countries' confidence in the US support and by pandering to the US domestic audience with a harsh demand.

Proposition 3 (Separating 3)**.** *When assumption 1 and Line 10, 11, 12, and 13 are satisfied and*

$$\overline{a} > (\underline{\beta}+2)a^* - \pi \tag{16}$$

$$1 > l > l^* \tag{17}$$

there exists a separating PBE at which the committed type of A offers $a = \overline{a}$, T rejects it, A does not withdraw from the alliance, C offers $x = p_h - c_T$, and T accepts it, and the uncommitted A offers $a = \overline{a}$, T rejects the offer, A withdraw from the alliance, C offers $x = p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 3 is a separating equilibrium where both the committed and uncommitted types of A offer the highest cost-sharing offer, $a = \overline{a}$, and the offer is rejected in both cases. The separation happens after the failure of the cost-sharing negotiation. The committed A remains in the alliance and obtains a better division of the issue ($x = p_h - c_t$), whereas the uncommitted A withdraws from the alliance and secures a very minimum division of the issue ($x = p_l - c_T$).

Proposition 3 is an equilibrium of alliance termination due to extremely high isolation-

ism. The strategy for the committed A is the same as Proposition 2: her strong commitment fails the cost-sharing negotiation in any case, so she panders to domestic isolationism and does not withdraw from the alliance after the failure of the negotiation. But the uncommitted type of A behaves differently from Proposition 2. As Line 17 suggests, her extremely strong isolationism makes her offer a very high cost-sharing offer, even though it could achieve a successful cost-sharing negotiation with lower demand like Proposition 2. This is because the benefit from extreme domestic isolationism dominates the benefit from successful negotiation and the sustainment of the alliance. By offering $a = \overline{a}$, the uncommitted type gets $l\overline{a}$, and since the value of l is high, the benefit from domestic politics is large. This large domestic benefit justifies the deficit of the alliance termination and the lower division of the issue. This is possible because of the high maximum ability to pay of T (Line 17). This equilibrium shows that, although the committed type can sustain the alliance even after the failure of the negotiation, the cost-sharing cannot save the alliance when domestic isolationism is extremely strong and the patron is not committed.

Proposition 4 (Pooling 1)**.** *When assumption 1, Line 10, 11, and 12 are satisfied and*

$$l < 0 \tag{18}$$

$$\pi < \underline{\beta}(p_h - p_l) \tag{19}$$

$$d < d^* = \frac{p_m - p_l}{p_m - p_l + c_T + c_C}, \tag{20}$$

where d is C's belief that A is the committed type after withdrawal, there exists a pooling PBE at which both types of A do not make a cost-sharing offer, C offers $p_h - c_T$, T accepts it, C offers $p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 5 (Pooling 2)**.** *When assumption 1 and Lines 10, 11, 12, 19, and 20 are*

satisfied and

$$l > 0 \tag{21}$$

there exists a pooling PBE at which both types of A offer $a = \overline{a}$, T rejects, A does not withdraw from the alliance, C offers $p_h - c_T$, and T accepts. See Appendix for proof.

Proposition 4 and 5 show two similar pooling equilibria where both types of A sustain the alliance due to a very small value of π (Line 19)[12]. Because of the small costs of the alliance, both types of A cannot make a credible threat of abandonment, and T does not accept any cost-sharing offer. Given this, both types of A make "no offer" to avoid domestic punishment when her constituencies are internationalists in Proposition 4 (See Line 18) or make the largest demand to benefit from domestic rewards when they are isolationists in Proposition 5 (See Line 21). Proposition 4 is what people usually think of as a normal alliance: The alliance continues to exist without any conflicts over cost-sharing. Proposition 5 is an equilibrium of uninformative cost-sharing demands. Unlike Proposition 2, a high cost-sharing demand does *not* update C's belief due to the identical pre-crisis-bargaining behavior of A.

6 Empirical Records

6.1 Japan in 1978

In 1978, Japan started shouldering the costs of its alliance with the US outside of the original framework of the alliance treaty. In the 1960 agreement between Japan and the

[12] As Line 20 shows, A's strategy is also supported by C's off-equilibrium-path belief that the withdrawal from the alliance is a sign of the uncommitted of A. This off-equilibrium-path belief passes the Intuitive Criterion (Cho and Kreps, 1987)

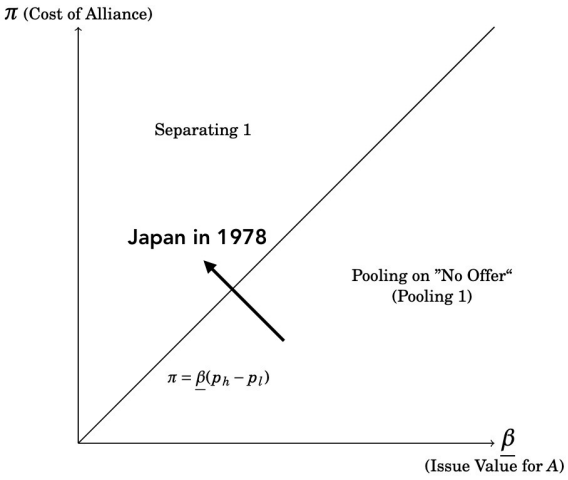

Figure 2: Key Equilibria of the Model

US (Japan Status of Forces Agreement (SOFA)), which is still effective in 2024, it is stipulated that the US should "bear...all expenditures incident to the maintenance of the United States armed forces in Japan" except for furnishing and making compensation of related facilities.[13] Nonetheless, Japan started paying some of the costs of the US bases in Japan outside of the agreement, including salaries of workers in US bases, utility bills, and construction costs for US facilities such as houses and amusement facilities. Since then, Japan's cost-sharing has increased over the years, and Japan's cost-offset percentage reached 74.5% of the total costs of US bases in Japan in 2002 according to the Department of Defense's report.[14]

I argue that 1978 was a key moment for the cost-sharing of Japan and that Japan entered the first separating equilibrium (Proposition 1) from the first pooling equilibrium (Proposition 4) in 1978. Following the advice by Goemans and Spaniel (2016), I offer three

[13] Article XXIV of the Status of Forces Agreement (SOFA) between the United States and Japan

[14] the Department of Defense, USA. "2004 Statistical Compendium on Allied Contributions to the Common Defense". (URL: https://apps.dtic.mil/sti/citations/ADA475431. Last Access: August 17, 2024)

sets of evidence. In the model, actors' behavior on equilibrium moves from Proposition 4 to Proposition 1 when (a) the cost of the alliance (π) becomes higher for A and (b) the value of the issue becomes lower for the uncommitted A ($\underline{\beta}$) (See Figure 2 for graphical understanding[15]), which creates larger uncertainty over A's resolve (Line10). Also, one key strategic consideration in the separating equilibrium is that (c) T accepts the cost-sharing offer because it worries that A will withdraw from the alliance otherwise. I will show that the cost of the alliance for the US was dramatically increasing, that the value of the alliance was getting lower at that time, which induced larger uncertainty of the US commitment, and that the US administration had a strategy to threaten that it would withdraw from the alliance and it was probably credible for Japan.

First, I provide evidence that the cost of the alliance was getting higher for the US. Japan's economic growth made US bases in Japan more expensive due to weaker US dollars, inflation in Japan, and an increase in salaries for Japanese workers. The USD/JPY exchange rate surged from 1976 to 1978. In 1976, one dollar was 296 yen, but it was 206 yen in 1978. This means the value of US dollars against Japanese Yen became two-thirds in two years. In addition, inflation in Japan made it more expensive to sustain the US bases in Japan. The Oil Shock caused inflation, and the inflation rate from 1975 to 1978 in Japan was 23%. The minimum wage in Okinawa, where the majority of US bases are located, increased by 7% only in 1978. These factors surged the costs of US bases in Japan.[16]

Key people in the US and Japanese governments stated the impact of the increasing

[15]Note that the threshold between Proposition 1 and Proposition 4 is $\underline{\beta}(p_h - p_l)$ as shown in Line 10 and 19. This is a clear cut and means that actors *must* enter Proposition 1 (or 2) as β decreases and π increases before they enter other types of equilibrium.

[16]In addition, the inflation in the 1980s in the US would have increased the domestic pressure to cut down the expenses of US bases overseas.

costs of US bases in Japan. In the Japanese diplomatic archive on a Japan-US meeting on March 21, 1977, President Carter was reported to state that the personal costs (salaries for workers) of the US forces stationed in Japan increased and this was a problem in the US Congress, so he would appreciate it if the prime minister of Japan considers Japan's sharing in this issue in the future[17]. Shin Kanemaru, who took the initiative in the new decision of the Japanese government as a Director General of the Japan Defense Agency at that time, recalled his memory that on April 4, 1978, Commander George Loving came and said that strong Yen and weak Dollars were causing trouble and he wanted Shin Kanemaru to do something (Kanemaru, 1979, pp. 78-79). All of these things show that the costs of the US-Japan alliance were getting expensive for the US, suggesting a high value in π.

Second, the value of the disputed issue decreased in 1978. The 1970s was an era of US withdrawal from Asia, evidenced by the improved US-China relationship, the bitter defeat in Vietnam, and a series of withdrawal plans from Asian countries, such as Thailand, Taiwan, and South Korea. Nixon Doctrine in 1969 stipulated the US grand strategy in the next decade. After Nixon visited China in 1972, it was evident that the US was seeking to improve its relationship with China. Under the Carter administration, the US-China (PRC) relationship became normalized and they announced that they would establish a formal diplomatic relationship in December 1978[18]. As a result, the US terminated its alliance with Taiwan[19] and withdrew its military forces from the country in 1979. China is not

[17]Telegram from U.S.A. to Tokyo, No.1286 (March 22, 1977), 2014-4987, Diplomatic Archives of the Ministry of Foreign Affairs Japan

[18]Website of Office of the Historian, *China Policy* (URL: https://history.state.gov/milestones/1977-1980/china-policy. Last Access: August 16th, 2024)

[19]The full name of the treaty is the Mutual Defense Treaty between the United States and the Republic of China, which was signed in 1954. This treaty requires Taiwan and the US to protect each other when either of them is attacked, but the regional application of the treaty is limited to mainland Taiwan and Pescadores, and Kinmen and Matsu islands were not included.

the only example. In 1975, the Vietnam War ended, in which US bases in Japan were used as bases for fighters and other aircrafts. This decreased the necessity of US bases in Japan. The end of the Vietnam War pushed the US to withdraw its military forces from Thailand in 1976. In addition, Carter won the presidential election in 1976 with a promise that the US would withdraw from South Korea. This plan was not carried out in the end, but, coupled with the US plan to pivot to NATO, it spread a question of US commitment to East Asia (Green, 2017, p. 377), suggesting larger uncertainty over the US's intentions. The Japanese side recognized the US's reluctant attitude. In a section titled "Does the US not help Japan any longer?" in his autobiography, Shin Kanemaru wrote that the structure of the US-Japan alliance was changing and that, with the withdrawal of the US forces from East Asia, "we can not deny that the importance of military intervention (for the US) will be decreasing in these regions"(Kanemaru, 1979, p. 76). All of these things suggest that it was worried that the benefit of sustaining US bases in East Asia was decreasing for the US due to the disappearance of China as an enemy and the disappearance of the battlefield in Vietnam, which were clarified by a series of withdrawal plans from Asian countries. This implies that the value of the issue at stake ($\underline{\beta}$) for the US was decreasing in 1978 and that this created larger uncertainty of the US's commitment, satisfying an important condition of Proposition 1 (Line 10 and 11).

Finally, an important strategic interaction in Proposition 4 is that the threat of withdrawal is credible. Although the US repeatedly tried to convince that the US would not abandon Japan and the Japanese government acknowledged that point in official meetings[20], some key people on the Japanese side were still skeptical. A Japanese historian points out that

[20]This must have contributed to the larger uncertainty as well.

reduction plans of US Marines in Okinawa and its new rotation routine caused concern for the US's future presence in Japan (Nozoe, 2014). People at the Japan Defense Agency were reported to say that they needed to think that the US land power would disappear in the Far East in the 1980s and that the withdrawal of the US Marines in Okinawa was a matter of time (Nozoe, 2014). Green (2017, p. 380) describes the US-Japan relationship at that time as follows. "(T)he U.S.-Japan alliance had been drifting since the Nixon shocks, and there was growing evidence that Tokyo might be hedging for the possibility of a post-American Pacific order", suggesting that Japan was preparing for the US's additional withdrawal from the region, where Japan was the only country that remained.

In fact, there is suggesting evidence that the US government exploited Japan's worry about the US commitment to induce concession from the Japanese side. On June 19, 1978, Mike Armacost, a member of the National Security Council in the Carter administration, gave pieces of advice to Zbigniew Brzezinski, an Assistant to the President for National Security Affairs, about his upcoming meeting with Shin Kanemaru. In the memorandum, Mike Armacost stated, "(Y)ou should...point out the impact of escalating costs on the willingness of the services to maintain forces in Japan" and "that a sizeable GOJ cost-sharing package would have enormous value in solidifying our security and political relationship [21]." This clearly shows that the US had a strategy to achieve a concession from Japan by threatening that Japan's cost-sharing would affect the continuation of US bases in Japan. Coupled with the US withdrawal from other Asian countries and Japan's worry that the same thing would happen in Japan soon, this threat must have been credible for

[21] Memorandum 3766 from Mike Marmacost to Zbigniew, Brzezinski's Country Files, Records of the Office of the National Security Advisor (Carter Administration), Box 40, Japan, 6-12/78. Jimmy Carter Library.

Japan. This is exactly what the model predicts as conditions of successful cost-sharing.

6.2 Japan in 2019

Japan in 2019 is another example of cost-sharing negotiations between allies. It is famous that Donald Trump was not satisfied with Japan's cost-sharing for the US-Japan alliance, even before his presidency (Sanger and Haberman, 2016). In November 2019, it was reported that the Trump administration asked Tokyo to pay four times as much to offset the costs of US forces in Japan (Seligman and Gramer, 2019), which is $8 billion according to John Bolton, former National Security Advisor (Bolton, 2024, Ch.11). The cost-sharing agreement effective at that time would expire in March 2021. So, Japan and the US officially started the negotiation around October 2020 to discuss the next 5-year cost-sharing, although the substantive negotiations between practitioners arguably began by July 2019 (Bolton, 2024, Ch.11). However, after a series of negotiations, time was run out and it was announced that the Trump administration and Japan gave up reaching an agreement (Tajima, 2020). Why did the US and Japan fail to reach a cost-sharing agreement?

Although details about the negotiation have not been disclosed yet, Proposition 2 provides an important interpretation of the failure of the cost-sharing negotiation between the Trump administration and Japan. Proposition 2 tells us that the failure of a cost-sharing negotiation happens when a patron is committed ($\overline{\beta}$), the threat of abandonment is not credible, and the domestic constituencies are isolationists but not too much, as Line 15 requires. In such a situation, the proposition says that a patron mainly tries to satisfy her domestic isolationism, which leads to a very high cost-sharing demand ($a = \overline{a}$). To prove the usefulness of the model, I empirically indicate that actors' important strategic

interactions in Proposition 2 actually happened in Japan in 2019. Specifically, I show that (a) Trump tried to use a threat of withdrawal to achieve a concession, (b) this threat was probably not credible for Japan because of the high resolve ($\overline{\beta}$), and (c) Trump was mainly concerned with domestic politics, where isolationism was strong enough ($l > 0$).

First, Trump was aware of a threat of withdrawal as a strategy to induce a concession from Japan. In his tell-all book, Bolton writes that "(H)e (Trump) said, as he did more and more frequently, that the way to get the $8 and $5 billion annual payments, respectively (Japan and South Korea), was to threaten to withdraw all US forces" (Bolton, 2024, Ch.11). Trump added, "That puts you in a very strong bargaining position" (Bolton, 2024, Ch.11). This suggests that Trump understood the threat of withdrawal as one of his cards at hand, which is an important moving part of the formal model. There is little doubt that Trump was thinking of the withdrawal as an actual polity option (Jacobs, 2019), but at the same time, he used his position as leverage to achieve concession.

Second, the threat of abandonment was probably not credible for Japan because of the US's strong commitment to Japan's security ($\overline{\beta}$). Whether Trump was serious about withdrawing from Japan was an important question for the Japanese government. Although Japan was probably not completely sure about what Trump was thinking, there is suggestive evidence that Japan is relatively confident in US support. Japan's optimism came from the good personal relationship between Trump and Shinzo Abe, the Prime Minister of Japan at that time, and the continuation (or possibly improvement) of US security policy for Japan in the Trump administration.

It is well known that Abe and Trump had a honeymoon relationship, as evidenced by the fact that they "met 20 times, played 5 rounds of golf, and had 32 phone calls, at times

speaking twice a week“ (Teraoka, 2020). Bolton (2024, Ch.11) describes their relationship as follows, “In my view, Trump’s best personal relationship among world leaders was with Abe (golf buddies as well as colleagues).” Abe himself recalls his relationship with Trump, “Trump had no problem with calling me for an hour, or an hour and a half when it was long...I tried to honestly tell my thoughts to Trump, and I believe that Trump told me his true thoughts on many issues as well.” (Abe et al., 2023, Ch.6)[22]. Abe’s investment in the personal relationship with Trump was fruitful to the extent that he successfully “convince(d) the United States to support his strategy in the Indo-Pacific, while drawing attention to issues of importance to Japan” (Teraoka, 2020), implying that the personal relationship was influential enough to pin down Japanese security environment in Trump’s mind.

Based on this personal relationship, the Trump administration often confirmed its support for Japan. In February 2017, soon after his inauguration, Trump said after a meeting with Abe, “This administration is committed to bringing those ties even closer. We are committed to the security of Japan and all areas under its administrative control and to further strengthening our very crucial alliance” (Hirschfeld and Baker, 2017). This commitment did not change after two years, when Trump became more critical of Japan. In a meeting in Osaka, “Abe and Trump reconfirmed that the nations’ long-standing cooperation on security arrangements remain in place” (Hara, 2019). The continuation of US security policy in the Trump administration may be caused by bureaucratic constraint to some extent (Drezner, 2019), but Trump’s strong commitment appears even when he personally criticized Japan: Trump condemned Japan by saying “If Japan is attacked, we

[22]Translated by the author

will fight World War Three...We will fight at all costs, right? But if we're attacked, Japan doesn't have to help us at all. They can watch it on a Sony television", suggesting that the US is ready to protect its ally in the Far East if necessary (Sieg and Leussink, 2019).

The Japanese government's evaluation of the Trump administration can be seen in an article written by an anonymous foreign ministry official. The article admits that "it (Trump's use of economic leverage against its allies) raised doubts in many minds across the region as to the credibility of American security guarantees and commitments" (this suggests the larger uncertainty for a third actor, which makes Line 10 and 11 satisfied more likely), but the article also claims that Japan surely prefers Trump's security polity to Obama's one by saying "do we want...to go back to the world before Trump? For many decision-makers in Tokyo, the answer is probably no" (Y.A., 2020), implying the continuation (or possibly improvement) of the US commitment in Japan in the Trump era. The Japanese people seemed to recognize this point as well, as shown by 82% and 67% of people expressing their confidence in US support in a war with North Korea and China, respectively, a seven-point increase from the Obama years in the latter number[23].

All of these things suggest Trump's high resolve for Japanese security ($\overline{\beta}$) and Japan's confidence in Trump's support. Proposition 2 tells us that the strong resolve creates an incentive for an ally not to accept any additional cost-sharing since the ally can benefit from the alliance without paying such costs. This is exactly what happened in reality, as shown by the failure of the cost-sharing negotiation in 2019. Moreover, this view provides an answer to the question of why Trump did not reduce his cost-sharing demand to make it easy for Japan to accept. Given that the next presidential election was getting closer and

[23] See (Green, 2022, p.102) for more on this.

Trump often criticized Japan's free-riding on the alliance, it was a good chance to appeal his competence. The model explains that Japan's confidence in Trump's strong support vanished Japan's incentive to pay *any* extra money, making lowering the cost-sharing demand meaningless.

Finally, Proposition2 states that when the threat of withdrawal is not credible, a patron tries to satisfy domestic isolationism with a very high demand. Empirical evidence suggests that domestic isolationism was strong among Trump supporters and Trump was mainly concerned with domestic politics. Pew Research Center reports that "55% of conservative Republicans say the U.S. should follow its own national interests even when allies disagree" in 2019 (Doherty et al., 2019), which is good evidence that domestic constituencies, especially Trump supporters, are isolationists on average at that time ($l > 0$)[24]. But, at the same time, this number also shows that domestic isolationism was not extremely high, as required by Line 15.

There is plenty of evidence that Trump's foreign policy attitude was targeting these domestic supporters. Many scholars would agree that Trump is a populist (see, for example, Boucher and Thies (2019) and Hafner-Burton et al. (2019)), which is featured with anti-elitism and the general will of the "pure" people. Boucher and Thies (2019) empirically shows that Trump's populist rhetoric created a unique cluster in social media and divided general people and elites in foreign policy issues. Moreover, Bolton said in an interview with a Japanese newspaper as follows, "Trump's yardstick was "how it affects domestic politics", even in national security issues. If Trump ever decided correctly, this

[24]To wit, McDonald et al. (2019) empirically shows that citizens do not care about politicians' consistency as much as they are often assumed and do not punish politicians who change their minds, especially in the Trump era. This suggests that Trump's partisanship makes it difficult to generate audience costs or sunk costs, implying that the model setting about domestic politics parameter (l) captures this feature well.

is not because his advisors successfully convinced him but because Trump worried that Republican politicians would oppose otherwise" (Sawamura, 2020)[25]. This clearly states that the main driver of Trump's foreign policy was domestic politics, not international politics.

In such a situation, Proposition 2 predicts that a patron demands the largest amount of cost-sharing, which actually happened in 2019. As explained above, the Trump administration demanded that Japan pay four times as much, which is about $8 billion. This number did not come from a realistic calculation, as shown Bolton (2024)'s recall that "only Trump knew what payment would satisfy him, so there was no point now trying to guess what the "real" number was. Trump himself didn't know yet." This is consistent with Proposition 2 in which a committed patron demands the largest amount of cost-sharing to satisfy domestic isolationism ($a = \overline{a}$), not to achieve international concessions.

7 Conclusion

Alliance cost-sharing is puzzling based on the widely -accepted understanding that alliances are for costly signaling (Morrow, 1994; Smith, 1995). To explain such a puzzle, I constructed a formal model in which allies negotiate their cost-sharing under the shadow of international crisis and domestic politics.

The model reveals that a credible threat of withdrawal from an alliance is key to successful cost-sharing negotiations. Moreover, the model identifies two means by which cost-sharing negotiations bring peace and deterrence. First, successful negotiations signal that a patron is not committed, but it also keeps the patron's involvement by reducing the

[25]Translated by the author.

alliance costs. This sustains an alliance capability boost and thus leads to deterrence and peace. To keep a patron's involvement, its protégé has to pay the exact value the protégé can save thanks to the alliance, implying a lower final payoff for the protégé. Second, a large cost-sharing demand makes the negotiations fail, but it signals the patron's commitment and panders to domestic isolationism at the same time. When a patron is committed, a threat of withdrawal is not credible, leading to the failure of cost-sharing negotiations. This failure shows a protégé's confidence in its patron's support, signaling the strong commitment of the patron. The patron makes a large cost-sharing demand in such a situation to get some benefit from pandering to her domestic isolationism. This suggests a *positive* association between a patron's strong commitment and a large cost-sharing demand. Empirical records of cost-sharing negotiations in the US-Japan alliance in 1978 and 2019 show the usefulness of the model. In the end, this article shows that both successful and failed cost-sharing negotiations are beneficial for deterrence in equilibria but in different ways. Cost-sharing negotiations work as a regulator valve both when a patron is committed and not committed, and thus, cost-sharing negotiations allow alliances to survive longer by overcoming changes in the strategic environment.

Some final notes on counterfactual scenarios would be useful. In this article, the success and failure of cost-sharing negotiations endogenously happen on equilibrium, meaning that exogenously changing actors' decisions may not work as intended. For example, rejecting a patron's offer when the patron is not committed would not automatically signal the patron's commitment. Rather it would trigger the withdrawal from the alliance, causing a bad consequence. Similarly, accepting a cost-sharing demand would not help an alliance survive longer when a patron is already committed. Rather, it decreases the protégé's

payoff for nothing. This article does not make any policy recommendations without careful thought and a correct understanding of the model.

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Appendix

Proof for Proposition 1

First of all, I analyze the crisis-bargaining stage by using backward induction. A helps T in a war if

$$\beta(p_h - c_A) + la > \beta(p_l - c_A)$$
$$\Leftrightarrow \beta(p_h - p_l) > c_A$$

when A has an alliance and

$$\beta(p_m - c_A) + la > \beta(p_l - c_A)$$
$$\Leftrightarrow \beta(p_m - p_l) > c_A$$

when A does not have an alliance. Thus, the committed A helps T regardless of the alliance but the uncommitted A helps T iff it has an alliance when

$$\min\{\overline{\beta}(p_m - p_l), \underline{\beta}(p_h - p_l)\} > c_A > \underline{\beta}(p_m - p_l)$$

, which is Line 8 in Assumption 1.

Given this incentive of A, T accepts the offer of the pie iff

$$x \geq p_h - c_T$$

, when A is committed or when A is not committed but has an alliance.

$$x \geq p_l - c_T$$

, when A is not committed and does not have an alliance. To make sure that there is a range of $x > 0$, we need to assume

$$p_l > c_T$$

, which is Line 7 in Assumption 1.

Because of the inefficiency of war, C's offer of x is $x = p_h - c_T$ when A and T have an alliance. When they do not have an alliance, C offers $x = p_m - c_T$ if C is sure that A is committed and $x = p_l - c_T$ if A is not committed.

Next, I move on to the analysis in the cost-sharing negotiation stage. With the equilibrium behavior of actors in Proposition 2 in mind, I first analyze A's decision to remain in or withdraw from the alliance after T's rejection of the cost-sharing offer.

If A remians in the alliance, C offers $x = p_h - c_T$ and T accepts. Thus,

$$u_A(\text{stay}) = \beta(p_h - c_T) - \pi + la$$

If A withdraws from the alliance, C thinks A is uncommitted and offers $x = p_l - c_T$. T

with the committed A rejects the offer and A with the uncommitted A accepts the offer.

$$u_A(\text{withdraw}) = \begin{cases} \overline{\beta} p_m - c_A + la & (\text{When } A \text{ is committed}) \\ \underline{\beta}(p_l - c_T) + la & (\text{When } A \text{ is not committed}) \end{cases}$$

When A is committed, A remians in the alliance iff

$$\begin{aligned} &\beta(p_h - c_T) - \pi + la > \overline{\beta} p_m - c_A + la \\ &\Leftrightarrow \overline{\beta}(p_h - p_m - c_T) + c_A > \pi \end{aligned} \tag{22}$$

When A is not not committed, A withdraws from the alliance iff

$$\begin{aligned} &\underline{\beta}(p_h - c_T) - \pi + la > \underline{\beta}(p_l - c_T) + la \\ &\Leftrightarrow \pi > \underline{\beta}(p_h - p_l) \end{aligned} \tag{23}$$

Next, I analyze T's incentive to accept the cost-sharing offer.

When T's partner A is committed,

$$u_T(\text{accept}) = p_h - c_T - a$$
$$u_T(\text{reject}) = p_h - c_T$$

T does never accept the offer since $a > 0$.

When T's partner A is not committed,

$$u_T(\text{accept}) = p_h - c_T - a$$
$$u_T(\text{reject}) = p_l - c_T$$

T accepts iff

$$\begin{aligned} & u_T(\text{accept}) > u_T(\text{reject}) \\ \Leftrightarrow\ & a^* = p_h - p_l \geq a \end{aligned} \tag{24}$$

Finally, A's optimal cost-sharing offer is analyzed.

When A is committed, any offer of cost-sharing is rejected. Thus, we get

$$u_A(\text{no offer}) = \overline{\beta}(p_h - c_T) - \pi$$
$$u_A(\text{offer } a) = \overline{\beta}(p_h - c_T) - \pi + la$$

Given that $\overline{\beta}(p_h - c_T) - \pi$ is the same in both payoffs, la determines A's action. A makes "no offer" when

$$l < 0 \tag{25}$$

A offers $a = \overline{a}$ because A gets the largest payoff at the largest a when

$$l > 0 \tag{26}$$

When A is not committed and T's maximum ability to pay is larger than a^* ($\overline{a} > a^*$), A's payoffs are expressed as follows.

$$\begin{aligned}
u_A(\text{no offer}) &= \underline{\beta}(p_h - c_T) - \pi \\
\arg\max_a u_A(\text{offer } a \le a^*) &= u_A(\text{offer } a = a^*) \\
&= \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \ (\because -1 < l < 1) \\
\arg\max_a u_A(\text{offer } a > a^*) &= \arg\max_a(\underline{\beta}(p_l - c_T) + la) \\
&= \begin{cases} u_A(\text{offer } a = a^* + \epsilon) \approx \underline{\beta}(p_l - c_T) + la^* \text{ when } l < 0 \\ u_A(\text{offer } a = \overline{a}) = \underline{\beta}(p_l - c_T) + l\overline{a} \text{ when } l > 0 \end{cases}
\end{aligned}$$

, where ϵ is a positive tiny value.

A offers $a = a^*$ when

$$u_A(\text{offer } a = a^*) > u_A(\text{no offer})$$
$$\Leftrightarrow a^*(l+1) > 0$$

, which is always true since $l > -1$, and

$$u_A(\text{offer } a = a^*) > u_A(\text{offer } a > a^*)$$
$$\Leftrightarrow \begin{cases} (\underline{\beta}+1)a^* > \pi \text{ when } l < 0 \\ \dfrac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > l \text{ when } l > 0 \end{cases}$$

Thus, when $l < 0$, A offers a^* if

$$(\underline{\beta}+1)a^* > \pi \tag{27}$$

and when $l > 0$, A offers a^* if

$$\min\{1, l^* = \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*}\} > l \tag{28}$$

When A is not committed and T's maximum ability to pay is smaller than a^* ($\overline{a} < a^*$),

any offer of cost-sharing is accepted by T. Thus, A's payoffs are expressed as follows.

$$
\begin{aligned}
u_A(\text{no offer}) &= \underline{\beta}(p_h - c_T) - \pi \\
\arg\max_a u_A(\text{offer } a) &= \arg\max_a (\underline{\beta}(p_h - c_T) - \pi + a + la) \\
&= \underline{\beta}(p_h - c_T) - \pi + \overline{a} + l\overline{a} \ (\because -1 < l)
\end{aligned}
$$

$$
\begin{aligned}
u_A(\text{offer } \overline{a}) &> u_A(\text{no offer}) \\
\overline{a}(l+1) &> 0
\end{aligned}
$$

, which is always true since $l > -1$. Thus, A always offers $a = \overline{a}$.

To sum up, Proposition 1, where the committed A does not offer in the cost-sharing negotiation and the uncommitted A offers $a = \min\{a^*, \overline{a}\}$ and T accepts, happens when Line 22, 23, 25, and 27 are satisfied.

Line 22 is more strict than Line 27 because

$$
\begin{aligned}
(\underline{\beta}+1)a^* &> \overline{\beta}(p_h - p_m - c_T) + c_A \\
\Leftrightarrow (\underline{\beta}+1)(p_h - p_l) - \overline{\beta}(p_h - p_m - c_T) &> c_A \qquad (29)
\end{aligned}
$$

Given the range of c_A in Line 8 in Assumption 1, we need to show that the maximum value of c_A is smaller than the left-hand side of Line 29.

$$
\begin{aligned}
&(\underline{\beta}+1)(p_h-p_l)-\overline{\beta}(p_h-p_m-c_T)>\overline{\beta}(p_m-p_l)\\
&\Leftrightarrow (p_h-p_l)(\underline{\beta}+1-\overline{\beta})>-\overline{\beta}c_T
\end{aligned}
$$

$$
\begin{aligned}
&(\underline{\beta}+1)(p_h-p_l)-\overline{\beta}(p_h-p_m-c_T)>\underline{\beta}(p_h-p_l)\\
&\Leftrightarrow p_h-p_l>0
\end{aligned}
$$

Both of them are true for sure because the LHS is positive but the RHS is negative or zero. Thus, Line 22 is more strict than Line 27.

Therefore, the set of conditions for Proposition 1 are Line 22, 23, and 25, which are

$$
\begin{aligned}
&l<0\\
&(\underline{\beta}+1)a^*>\overline{\beta}(p_h-p_m-c_T)+c_A>\pi>\underline{\beta}(p_h-p_l)
\end{aligned}
$$

Finally, we need to check the existence of π.

$$
\begin{aligned}
&\overline{\beta}(p_h-p_m-c_T)+c_A>\underline{\beta}(p_h-p_l)\\
&\Leftrightarrow \overline{\beta}>\frac{\underline{\beta}(p_h-p_l)-c_A}{p_h-p_m-c_T}
\end{aligned}
\tag{30}
$$

Then, let's check if this satisfies $1>\overline{\beta}>0$.

The numerator $\underline{\beta}(p_h-p_l)-c_A$ is positive because of the condition of c_A (Line 8 in Assumption 1). So, in order to satisfy $\overline{\beta}>0$, the numerator must be positive.

$$p_h - p_m - c_T > 0$$

$$p_h - p_m > c_T \tag{31}$$

In order to satisfy $1 > \overline{\beta}$, we need to make sure the following condition is true.

$$\begin{aligned} 1 &> \frac{\underline{\beta}(p_h - p_l) - c_A}{p_h - p_m - c_T} \\ \Leftrightarrow \frac{p_h - p_m - c_T + c_A}{p_h - p_l} &> \underline{\beta} \end{aligned} \tag{32}$$

This is in the range of $0 < \underline{\beta} < 1$. First, this satisfies $\underline{\beta} > 0$ since the numerator is positive thanks to $p_l - p_m > c_T$ (Line 31) and the denominator is also positive for sure ($p_h > p_l$). Second, this satisfies $\underline{\beta} < 1$ because

$$\begin{aligned} \frac{p_h - p_m - c_T + c_A}{p_h - p_l} &< 1 \\ \Leftrightarrow p_m - p_l + c_T &> c_A \\ \Leftrightarrow p_m - p_l + c_T > \overline{\beta}(p_m - p_l) &> c_A \end{aligned}$$

This is sure to be true because of the upper limit of c_A in Line 8 in Assumption 1.

Therefore, Proposition 1 exists when Line 22, 23, 25, 30, 31, and 32 are satisfied.

Proof for Proposition 2

.

Proposition 2 is a separating equilibrium where the committed A offers $a = \overline{a}$, T rejects it, A does not withdraw from the alliance, the uncommitted A offers $a = a^*$, and T accepts it.

Most of the proof for this equilibrium is offered in the previous section. The committed A does not withdraw from the alliance after T's rejection of the cost-sharing offer when Line 22 is satisfied and the uncommitted A withdraws from the alliance when Line 23 is satisfied. π exists between these two conditions when Line 30, 31, and 32 are met. T's strategy of acceptance or rejection does not change, either. The committed A offers $a = \overline{a}$ when Line 26 is satisfied. The uncommitted A offers $a = a^*$ when $\overline{a} > a^*$ and Line 28 are satisfied.

The final thing we need to prove is the existence of l in Line 28. $1 > l > 0$ is fine. The existence of l in $l^* = \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > l > 0$ should be checked.

$$
\begin{aligned}
& l^* > 0 \\
\Leftrightarrow & \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > 0 \\
\Leftrightarrow & (\underline{\beta}+1)a^* > \pi \quad (\because \overline{a} > a^*)
\end{aligned}
$$

The maximum value of π is $\overline{\beta}(p_h - p_m - c_T) + c_A > \pi$ as shown in Line 22, so we need to show that π's maximum value is smaller than $(\underline{\beta}+1)a^*$.

$$(\underline{\beta}+1)a^* > \overline{\beta}(p_h - p_m - c_T) + c_A$$
$$\Leftrightarrow (\underline{\beta}+1)(p_{h-p_l}) - \overline{\beta}(p_h - p_m - c_T) > c_A$$

This is sure to be true because of the range of c_A in the assumption (Line 8), as shown in Line 29.

Therefore, Proposition 2 exists when Line 22, 23, 26, 28, $\overline{a} > a^*$, 30, 31, and 32.

Proof for Proposition 3

.

Proposition 3 is a separating equilibrium where both the committed A and the uncommitted A offer $\overline{a}$, T rejects it, and the committed A does not withdraw from the alliance, whereas the uncommitted A withdraw.

Again, most of the proof is already shown in the previous sections. The committed A does not withdraw from the alliance after T's rejection of the cost-sharing offer when Line 22 is satisfied and the uncommitted A withdraws from the alliance when Line 23 is satisfied. π exists between these two conditions when Line 30, 31, and 32 are met. T's strategy of acceptance or rejection of the cost-sharing offer is also the same as the previous two equilibria. The committed A offers $a = \overline{a}$ when Line 26 is satisfied. The difference from previous equilibria is the cost-sharing offer by the uncommitted type.

When T's maximum ability to pay is larger than a^* ($\overline{a} > a^*$) and and $l > 0$ as Line 26 requires, the uncommitted A's payoffs are

$$
\begin{aligned}
u_A(\text{no offer}) &= \underline{\beta}(p_h - c_T) - \pi \\
\arg\max_a u_A(\text{offer } a \le a^*) &= u_A(\text{offer } a = a^*) \\
&= \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \ (\because -1 < l < 1) \\
\arg\max_a u_A(\text{offer } a > a^*) &= \arg\max_a (\underline{\beta}(p_l - c_T) + la) \\
&= u_A(\text{offer } a = \overline{a}) = \underline{\beta}(p_l - c_T) + l\overline{a}
\end{aligned}
$$

A offers $\overline{a}$ if

$$
\begin{aligned}
& u_A(\text{offer } a = \overline{a}) > u_A(\text{no offer}) \\
\Leftrightarrow\ & \underline{\beta}(p_l - c_T) + l\overline{a} > \underline{\beta}(p_h - c_T) - \pi \\
\Leftrightarrow\ & \pi > \underline{\beta}(p_h - p_l) - l\overline{a}
\end{aligned}
$$

, which is sure to be true because of the lower limit of π in Line 23 ($\pi > \underline{\beta}$), and

$$
\begin{aligned}
& u_A(\text{offer } a = \overline{a}) > u_A(\text{offer } a = a^*) \\
\Leftrightarrow\ & \underline{\beta}(p_l - c_T) + l\overline{a} > \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \\
\Leftrightarrow\ & l > l^* = \frac{(\underline{\beta} + 1)a^* - \pi}{\overline{a} - a^*}
\end{aligned} \tag{33}
$$

Because of Line 26, we should check if this is within the range of $0 < l < 1$. $l^* > 0$ is true as proved in the previous section. This means that Line 33 is more strict than 26.

$l^* < 1$ is also true when

$$\begin{aligned} & l^* < 1 \\ \Leftrightarrow\ & \overline{a} > (\underline{\beta}+2)a^* - \pi \end{aligned} \tag{34}$$

This condition is more strict than $\overline{a} > a^*$.

Therefore, Proposition 3 exists when Line 22, 23, 30, 31, 33, and 34.

Proof for Proposition 4

Proposition 4 is a pooling equilibrium where both the committed type and the uncommitted type do not make a cost-sharing offer. In the off-equilibrium path, T would reject any offers, and A does not withdraw from the alliance after the rejection. C believes that the probability that A is a committed type after its withdrawal from the alliance is less than d^*.

First of all, let's consider C's offer after A withdraws from the alliance. As discussed at the beginning of the proof for Proposition 1, C offers $x = p_m - c_T$ when A is committed and $x = p_l - c_T$ when A is not committed. When C is not sure about A's type, C has to decide to make a risky offer ($x = p_l - c_T$) or a safer offer ($x = p_m - c_T$). Let d be C's belief of $Pr(\overline{\beta}|A\text{'s withdrawal})$. The payoffs of each offer for C are

$$
\begin{aligned}
&u_C(x = p_m - c_T) = 1 - p_m + c_T \\
&u_C(x = p_l - c_T) = d(1 - p_m - c_C) + (1 - d)(1 - p_l + c_T) \\
&u_C(x = p_m - c_T) > u_C(x = p_l - c_T) \\
&\Leftrightarrow d > d^* = \frac{p_m - p_l}{p_m - p_l + c_T + c_C}
\end{aligned} \tag{35}
$$

Thus, C offers $x = p_m - c_T$ iff $d \geq d^*$ and $x = p_l - c_T$ iff $d < d^*$.

Hereafter, I assume that C's off-equilibrium-path belief that A is committed after she withdraws from the alliance is less than d^*, or $d < d^*$, which means that C offers $x = p_l - c_T$ after alliance termination.

With this belief in mind, let's consider actors' behavior in the cost-sharing negotiation stage.

First, if A remians in the alliance after T's rejection of her cost-sharing offer, C offers $x = p_h - c_T$ and this is accepted by T. A's payoff in such a situation is

$$
u_A(\text{stay}) = \beta(p_h - c_T) - \pi + la
$$

When A withdraws from the alliance, C offers $x = p_l - c_T$ as $d < d^*$. T having the committed A rejects this offer and war happens. T having the uncommitted A accepts the offer, and peace is sustained. Thus, A's payoffs of withdrawing from the alliance are

$$u_A(\text{withdraw}) = \begin{cases} \overline{\beta} p_m - c_A + la & (\text{When } A \text{ is committed}) \\ \underline{\beta}(p_l - c_T) + la & (\text{When } A \text{ is not committed}) \end{cases}$$

The committed A remians in the alliance when

$$\begin{aligned} & u_A(\text{stay}) > u_A(\text{withdraw}) \\ \Leftrightarrow\ & \overline{\beta}(p_h - p_m - c_T) + c_A > \pi \end{aligned} \tag{36}$$

The uncommitted A also remians in the alliance when

$$\begin{aligned} & u_A(\text{stay}) > u_A(\text{withdraw}) \\ \Leftrightarrow\ & \underline{\beta}(p_h - p_l) > \pi \end{aligned} \tag{37}$$

Line 37 is more strict than Line 36 when

$$\begin{aligned} \overline{\beta}(p_h - p_m - c_T) + c_A &> \underline{\beta}(p_h - p_l) \\ \Leftrightarrow \overline{\beta} &> \frac{\underline{\beta}(p_h - p_l) - c_T}{p_h - p_m - c_T} \end{aligned} \tag{38}$$

This satisfies $0 < \overline{\beta} < 1$ when

$$\frac{p_h - p_m - c_T + c_A}{p_h - p_l} > \underline{\beta} \tag{39}$$

$$p_h - p_m > c_T \tag{40}$$

(See Line 31 and 32 in Proof for Proposition 1)

Next, regardless of A's type, T does not have an incentive to accept any cost-sharing offer because

$$u_T(\text{accept}) = p_h - c_T - a$$
$$u_T(\text{reject}) = p_h - c_T$$
$$u_T(\text{reject}) > u_T(\text{accept})$$

The next step is to investigate the optimal level of A's cost-sharing offer. A's payoffs are expressed as follows.

$$u_A(\text{no offer}) = \beta(p_h - c_T) - \pi$$
$$u_A(\text{offer } a) = \beta(p_h - c_T) - \pi + la$$

A makes "no offer" when

$$l < 0 \tag{41}$$

A offers $a = \overline{a}$ because A gets the largest payoff when

$$l > 0 \tag{42}$$

To sum up, when Line 35, 37, 38, 39, and 40 are satisfied, both the committed A and

the uncommitted A do not withdraw from the alliance after the rejection by T. When Line 41 is satisfied, both types of A do not offer anything. Therefore, when these conditions are met, Proposition 4 exists.

Proof for Proposition 5

The necessary parts of the proof for Proposition 5 are shown in the previous section.

When Line 35, 37, 38, 39, and 40 are satisfied, both the committed A and the uncommitted A do not withdraw from the alliance after the rejection by T. When Line 42 is satisfied, both types of A offer $a = \overline{a}$. Therefore, when these conditions are met, Proposition 5 exists.